

Gauge invariant Lagrangian formulation of massive higher spin fields in $(A)dS_3$ space

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Abstract

We develop the frame-like formulation of massive bosonic higher spins fields in the case of 3-dimensional $(A)dS$ space with the arbitrary cosmological constant. The formulation is based on gauge-invariant description by involving the Stueckelberg auxiliary fields. The explicit form of the Lagrangians and the gauge transformation laws are found. The theory can be written in terms of gauge-invariant objects similar to the massless theories, thus allowing us to hope to use the same methods for investigation of interactions. In the massive spin 3 field example we are able to rewrite the Lagrangian using the new the so-called separated variables, so that the study of Lagrangian formulation reduces to finding the Lagrangian containing only half of the fields. The same construction takes places for arbitrary integer spin field as well. Further working in terms of separated variables, we build Lagrangian for arbitrary integer spin and write it in terms of gauge-invariant objects. Also, we demonstrate how to restore the full set of variables, thus receiving Lagrangian for the massive fields of arbitrary spin in the terms of initial fields.

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1 Introduction

The construction of the consistent theory describing the interacting dynamics of higher spin fields is an old and intriguing problem that really very far to be solved (for some recent reviews see [1]). Taking into account complexity of the problem and technical difficulties it seems that higher spin theories in three dimensions that appear to be much simpler may provide nice playground to gain some useful experience. In particular, contrary to the situation in $d \geq 4$ dimensions [2], in three dimensions it is not necessary to consider infinite number of massless higher spin fields to build a consistent interacting theory [3–5]. Also it is important that similar to gravity and supergravity theories in three dimensions [6], many higher spin models can be considered as Chern-Simons ones [4, 5] and such possibility of rewriting the action in the Chern-Simons form is widely used in investigations of possible higher spin interactions.

Till now most of works on the three dimensional higher spin theories were devoted to construction of interaction for massless fields [4, 5, 7] or parity odd topologically massive ones [8] with the most notable exception being the so called New massive gravity [9] (see also [10]). But massless higher spin fields in three dimensions being pure gauge do not have any physical degrees of freedom, while topologically massive one though do contain physical degrees of freedom but due to their specific properties can hardly be generalized to higher dimensions. That is why we think that it is important to investigate parity even interacting theories for massive higher spin fields. On the one hand, such theories will certainly have physical interest by themselves and from the other hand we may expect that they admit higher dimensional generalizations.

Lagrangian description for massive higher spin fields in three dimensions for the first time was given in [11]. In this paper we provide frame-like gauge invariant formalism for such fields, which in our opinion is the most suitable one for investigation of massive higher spin field interactions. Recall that frame-like formalism, generalizing the well known frame formalism for gravity, was initially formulated for massless fields [12] and then extended to massive case [13, 14]. Such formalism nicely works both in flat Minkowski space as well as in $(A)dS$ spaces with arbitrary value of cosmological constant. Also it gives a possibility to use for the massive fields the results obtained for more simple case of massless fields. Last but not least, it is the application of frame-like formalism that allows reformulating the massless three dimensional theories in the Chern-Simons form [5]. In this work we will pay special attention to the possibility to write Lagrangians for massive fields in terms of gauge invariant objects and the Chern-Simons actions.

The work is organized as follows. In the rest part of the introduction we formulate our notations and conventions as well as give basic information on the frame-like formulation of higher spin fields. In section 2 we briefly review the massless bosonic fields in the frame-like formulation confining ourselves by main results and the possibility to reformulate these theories in terms of separated variables only. In section 3 we consider in detail the example of massive spin 3 field. At first, we give explicit expressions for the Lagrangian and gauge transformations and then we introduce a gauge-invariant objects in terms of which Lagrangian is rewritten in the Chern-Simons form. Also we show that it is possible to introduce separated variables in a way similar to the massless case. In section 4 we consider massive fields with an arbitrary integer spin. This time, based on experience obtained in spin 3 case, we, from the very beginning, work with separated variables and the Lagrangian written in the Chern-Simons form so that construction turns out to be straightforward and rather simple. And then we easily obtain the complete theory in terms of initial fields.

Notations and conventions. In the frame-like formalism the fields of the arbitrary integer

spin are described by a pair of 1-forms¹

$$\Phi_\mu^{a_1 a_2 \dots a_{s-1}}, \quad \Omega_\mu^{b, a_1 a_2 \dots a_{s-1}}, \quad \Omega_\mu^{(b, a_1 a_2 \dots a_{s-1})} = 0$$

Here Greek letters are used for the world indices and Latin letters for the local ones. The fields are totally symmetric with respect to indices a and traceless in all local indices. Indices in parentheses here and below will mean a complete symmetrization without normalization factor. The first field generalizes the frame field in gravity and plays the role of the physical field and the second one is auxiliary generalizing the concept of the Lorentz connection in gravity. In $d = 3$ there is a fully antisymmetric third rank tensor ε^{abc} , using which we can introduce the dual field

$$\Omega_\mu^{a_1 a_2 \dots a_{s-1}} = \varepsilon^{bc(a_1} \Omega_\mu^{b, c|a_2 \dots a_{s-1})}$$

which is completely symmetric and traceless on local indices.

We work in the $(A)dS$ space and use the notation D_μ for covariant derivative normalized as follows

$$[D_\mu, D_\nu] \xi^a = \lambda^2 e_{[\mu}^a \xi_{\nu]}, \quad \lambda^2 = -\Lambda$$

where e_μ^a plays the role of (non-dynamical) frame of $(A)dS$ background, and Λ is an arbitrary cosmological constant. One can always consider the situation with a positive or negative Λ , as well as analyze the flat limit, putting $\lambda = 0$. Using the frame e_μ^a and its inverse, whenever it is convenient we convert local indices into world ones and back, for example

$$\Phi_{\mu, \nu}^{a_1 \dots a_{s-2}} = e_\nu^{a_{s-1}} \Phi_\mu^{a_1 \dots a_{s-1}}$$

We will write down the world indices explicitly, so to write the expression in totally antisymmetric form on the world indices (which is equivalent to an external product of 1-forms) we will often use the notation

$$\left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} = e^\mu_a e^\nu_b - e^\mu_b e^\nu_a$$

and similarly for $\left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\}$.

2 The kinematics of massless fields

In this section we give frame-like formalism for the massless fields in $(A)dS_3$ space which then will be used as building blocks in gauge invariant description for massive ones.

Spin 2 This particle is described by a pair of fields Ω_μ^a, f_μ^a with the following free Lagrangian

$$\mathcal{L} = \frac{1}{2} \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \Omega_\mu^a \Omega_\nu^b - \varepsilon^{\mu\nu\alpha} \Omega_\mu^a D_\nu f_\alpha^a + \frac{\lambda^2}{2} \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} f_\mu^a f_\nu^b \quad (1)$$

which is invariant under the following gauge transformations

$$\delta \Omega_\mu^a = D_\mu \eta^a + \lambda^2 \varepsilon_\mu^{ab} \xi^b, \quad \delta f_\mu^a = D_\mu \xi^a + \varepsilon_\mu^{ab} \eta^b \quad (2)$$

¹In $d \geq 4$ dimensions one also has to introduce so called extra fields $\Omega_\mu^{b_1 \dots b_t, a_1 \dots a_{s-1}}$, $2 \leq t \leq s-1$. But in three dimensions all these fields are absent and this greatly simplifies all calculations.

For all massless higher spin fields ($s \geq 2$) the free Lagrangians can be conveniently rewritten using gauge invariant objects. For $s = 2$ we introduce the dual curvature and torsion

$$\begin{aligned}\mathcal{F}_{\mu\nu}{}^a &= D_{[\mu}\Omega_{\nu]}{}^a + \lambda^2 \varepsilon_{[\mu}{}^{ab} f_{\nu]}{}^b \\ \mathcal{T}_{\mu\nu}{}^a &= D_{[\mu}f_{\nu]}{}^a + \varepsilon_{[\mu}{}^{ab}\Omega_{\nu]}{}^b\end{aligned}\quad (3)$$

With the help of these object the Lagrangian can be rewritten as

$$\mathcal{L} = -\frac{1}{4}\varepsilon^{\mu\nu\alpha}[\Omega_\mu{}^a\mathcal{T}_{\nu\alpha}{}^a + f_\mu{}^a\mathcal{F}_{\nu\alpha}{}^a]\quad (4)$$

In AdS_3 space with non-zero cosmological constant we can reformulate theory in terms of new separated variables

$$\Omega_{\pm\mu}{}^a = \Omega_\mu{}^a \pm \lambda f_\mu{}^a, \quad \eta_\pm{}^a = \eta^a \pm \lambda \xi^a\quad (5)$$

and corresponding gauge invariant objects

$$\mathcal{F}_{\pm\mu\nu}{}^a = \mathcal{F}_{\mu\nu}{}^a \pm \lambda \mathcal{T}_{\mu\nu}{}^a = D_{[\mu}\Omega_{\pm\nu]}{}^a \pm \lambda \varepsilon_{[\mu}{}^{ab}\Omega_{\pm\nu]}{}^b\quad (6)$$

Then the original Lagrangian in the new field variables has the form

$$\mathcal{L} = -\frac{1}{8\lambda}\varepsilon^{\mu\nu\alpha}[\Omega_{+\mu}{}^a\mathcal{F}_{+\nu\alpha}{}^a - \Omega_{-\mu}{}^a\mathcal{F}_{-\nu\alpha}{}^a]\quad (7)$$

Thus it is clear that the variables are separated and each half of the Lagrangian is invariant under its own gauge transformation of the same type, namely

$$\delta\Omega_{\pm\mu}{}^a = D_\mu\eta_\pm{}^a \pm \lambda \varepsilon_\mu{}^{ab}\eta_\pm{}^b\quad (8)$$

Moreover, in such formulation the invariance of the Lagrangian can be easily checked using differential identities for curvatures:

$$D_{[\mu}\mathcal{F}_{\pm\nu\alpha]}{}^a = \mp \lambda \varepsilon_{[\mu}{}^{ab}\mathcal{F}_{\pm\nu\alpha]}{}^b\quad (9)$$

Thus, we can work with one half of the Lagrangian and one field only.

Arbitrary spin s . Here we will use a pair of fields $\Omega_\mu{}^{a_1 a_2 \dots a_{s-1}}$ and $\Phi_\mu{}^{a_1 a_2 \dots a_{s-1}}$ which are completely symmetric and traceless in local indices. To simplify expressions we will use compact notations $\Phi_\mu{}^{a_1 \dots a_k} = \Phi_\mu{}^{(k)}$ and similarly for all other objects where k denotes the number of local indices. In these notations the free Lagrangian has the form

$$\mathcal{L} = (-1)^s \left[-\varepsilon^{\mu\nu\alpha}\Omega_\mu{}^{(s-1)}D_\nu\Phi_\alpha{}^{(s-1)} + \frac{s-1}{2}\{\begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix}\}[\Omega_\mu{}^{a(s-2)}\Omega_\nu{}^{b(s-2)} + \lambda^2\Phi_\mu{}^{a(s-2)}\Phi_\nu{}^{b(s-2)}] \right]\quad (10)$$

while gauge transformations look like

$$\begin{aligned}\delta\Omega_\mu{}^{(s-1)} &= D_\mu\eta^{(s-1)} - \lambda^2\varepsilon_\mu{}^{b(1}\xi^{s-2)b)} \\ \delta\Phi_\mu{}^{(s-1)} &= D_\mu\xi^{(s-1)} - \varepsilon_\mu{}^{b(1}\eta^{s-2)b)}\end{aligned}\quad (11)$$

As in the spin 2 case, the Lagrangian can be rewritten in terms of gauge invariant objects (which we will call curvatures)

$$\begin{aligned}\mathcal{G}_{\mu\nu}{}^{(s-1)} &= D_{[\mu}\Omega_{\nu]}{}^{(s-1)} - \lambda^2\varepsilon_{[\mu}{}^{b(1}\Phi_{\nu]}{}^{s-2)b)} \\ \mathcal{H}_{\mu\nu}{}^{(s-1)} &= D_{[\mu}\Phi_{\nu]}{}^{(s-1)} - \varepsilon_{[\mu}{}^{b(1}\Omega_{\nu]}{}^{s-2)b)}\end{aligned}\quad (12)$$

as

$$\mathcal{L} = -\frac{(-1)^s}{4}\varepsilon^{\mu\nu\alpha}[\Omega_\mu^{(s-1)}\mathcal{H}_{\nu\alpha}^{(s-1)} + \Phi_\mu^{(s-1)}\mathcal{G}_{\nu\alpha}^{(s-1)}] \quad (13)$$

By analogy with spin 2 case we introduce the new variables

$$\Omega_{\pm\mu}^{(s-1)} = \Omega_\mu^{(s-1)} \pm \lambda\Phi_\mu^{(s-1)} \quad (14)$$

in this, the Lagrangian can be rewritten as follows

$$\mathcal{L} = -\frac{(-1)^s}{8\lambda}\varepsilon^{\mu\nu\alpha}[\Omega_{+\mu}^{(s-1)}\mathcal{G}_{+\nu\alpha}^{(s-1)} - \Omega_{-\mu}^{(s-1)}\mathcal{G}_{-\nu\alpha}^{(s-1)}] \quad (15)$$

where

$$\mathcal{G}_{\pm\mu\nu}^{(s-1)} = \mathcal{G}_{\pm\mu\nu}^{(s-1)} \pm \lambda\mathcal{H}_{\pm\mu\nu}^{(s-1)} = D_{[\mu}\Omega_{\pm\nu]}^{(s-1)} \mp \lambda\varepsilon_{[\mu}^{b(1)}\Omega_{\pm\nu]}^{s-2)b} \quad (16)$$

Each half of the Lagrangian is invariant with respect to one type of gauge transformations

$$\delta\Omega_{\pm\mu}^{(s-1)} = D_\mu\eta_\pm^{(s-1)} \mp \lambda\varepsilon_\mu^{b(1)}\eta_\pm^{s-2)b}, \quad \eta_\pm^{(s-1)} = \eta^{(s-1)} \pm \lambda\xi^{(s-1)} \quad (17)$$

Similarly to spin 2 case, the invariance of the Lagrangian in such formulation directly follows from differential identities for curvatures:

$$D_{[\mu}\mathcal{G}_{\pm\nu\alpha]}^{(k)} = \pm\lambda\varepsilon_{[\mu}^{a(1)}\mathcal{G}_{\pm\nu\alpha]}^{k-1)a} \quad (18)$$

Thus as in the spin 2 case one can work with only one field and one half of the Lagrangian, which in the component form looks like:

$$\mathcal{L} = \frac{(-1)^s}{4\lambda}[\lambda(s-1)\{\begin{smallmatrix}\mu\nu\\ab\end{smallmatrix}\}\Omega_{+\mu}^{a(s-2)}\Omega_{+\nu}^{b(s-2)} - \varepsilon^{\mu\nu\alpha}\Omega_{+\mu}^{(s-1)}D_\nu\Omega_{+\alpha}^{(s-1)}] \quad (19)$$

In the massless case it may seem that using the separated variables does not simplify calculations very much. However, in a massive case due to the large number of fields involved such technics gives significant simplification.

3 Massive spin 3 field

In this section we begin with the usual gauge invariant description of massive spin 3 field adopted to $d = 3$ dimensions and then we show that it can be reformulated in terms of separated variables similar to the massless case.

Massive spin 3 field for the frame-like gauge invariant description [13] requires four pairs of fields $(\Omega_\mu^{ab}, \Phi_\mu^{ab})$, (Ω_μ^a, f_μ^a) , (B^a, A_μ) and (π^a, φ) , the last three of which play the roles of Stueckelberg fields. The Lagrangian has the form

$$\begin{aligned} \mathcal{L}_0 = & -\{\begin{smallmatrix}\mu\nu\\ab\end{smallmatrix}\}\Omega_\mu^{ac}\Omega_\nu^{bc} + \varepsilon^{\mu\nu\alpha}\Omega_\mu^{ab}D_\nu\Phi_\alpha^{ab} + \frac{1}{2}\{\begin{smallmatrix}\mu\nu\\ab\end{smallmatrix}\}\Omega_\mu^a\Omega_\nu^b - \varepsilon^{\mu\nu\alpha}\Omega_\mu^aD_\nu f_\alpha^a + \\ & + \frac{1}{2}B^aB^a - \varepsilon^{\mu\nu\alpha}B_\mu D_\nu A_\alpha - \frac{1}{2}\pi^a\pi^a + \pi^\mu D_\mu\varphi + \\ & + \varepsilon^{\mu\nu\alpha}[3m\Omega_{\mu,\nu}^a f_\alpha^a + m\Phi_{\mu,\nu}^a\Omega_\alpha^a - 2\tilde{m}\Omega_{\mu,\nu}A_\alpha + \tilde{m}f_{\mu,\nu}B_\alpha] + M\pi^\mu A_\mu + \\ & + \{\begin{smallmatrix}\mu\nu\\ab\end{smallmatrix}\}[-\frac{M^2}{36}\Phi_\mu^{ac}\Phi_\nu^{bc} + \frac{M^2}{8}f_\mu^a f_\nu^b] + M\tilde{m}e^\mu{}_a f_\mu^a \varphi + 3\tilde{m}^2\varphi^2 \end{aligned} \quad (20)$$

and is invariant under the following set of gauge transformations

$$\begin{aligned}
\delta_0 \Omega_\mu^{ab} &= D_\mu \eta^{ab} + \frac{a_2}{2} (e_\mu^{(a} \eta^{b)}) - \frac{2}{3} g^{ab} \eta_\mu - \frac{M^2}{36} \varepsilon_\mu^{c(a} \xi^{b)c} \\
\delta_0 \Phi_\mu^{ab} &= D_\mu \xi^{ab} - \varepsilon_\mu^{c(a} \eta^{b)c} + \frac{3m}{2} (e_\mu^{(a} \xi^{b)}) - \frac{2}{3} g^{ab} \xi_\mu \\
\delta_0 \Omega_\mu^a &= D_\mu \eta^a + 3m \eta_\mu^a + \frac{M^2}{4} \varepsilon_\mu^{ab} \xi^b \\
\delta_0 f_\mu^a &= D_\mu \xi^a + \varepsilon_\mu^{ab} \eta^b + m \xi_\mu^a + 2\tilde{m} e_\mu^a \xi \\
\delta_0 B^a &= -2\tilde{m} \eta^a, \quad \delta_0 A_\mu = D_\mu \xi + \tilde{m} \xi_\mu \\
\delta_0 \pi^a &= M\tilde{m} \xi^a, \quad \delta_0 \varphi = -M\xi
\end{aligned} \tag{21}$$

where

$$\tilde{m}^2 = 8m^2 + 4\lambda^2, \quad M^2 = 18(3m^2 + 2\lambda^2)$$

Contrary to the massless case this Lagrangian can not be rewritten in terms of separated variables. Indeed, while description of spin 3 and spin 2 components contains the symmetric pairs $(\Omega_\mu^{ab}, \Phi_\mu^{ab})$ and (Ω_μ^a, f_μ^a) , it is not the case for spin 1 and spin 0 ones. To make it possible we partially fix the gauge $\varphi = 0$, solve the algebraic equation for the π field expressing $A_\mu = \frac{1}{M} \pi_\mu$ and change the normalization $\pi_\mu \rightarrow M\pi_\mu$, because now the field π^a will play the role of physical one. As a result, we obtain the Lagrangian

$$\begin{aligned}
\mathcal{L}_0 &= -\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} \Omega_\mu^{ac} \Omega_\nu^{bc} + \varepsilon^{\mu\nu\alpha} \Omega_\mu^{ab} D_\nu \Phi_\alpha^{ab} + \frac{1}{2} \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} \Omega_\mu^a \Omega_\nu^b - \varepsilon^{\mu\nu\alpha} \Omega_\mu^a D_\nu f_\alpha^a + \\
&+ \frac{1}{2} B^a B_a - \varepsilon^{\mu\nu\alpha} B_\mu D_\nu \pi_\alpha + \\
&+ \varepsilon^{\mu\nu\alpha} [3m \Omega_{\mu,\nu}^a f_\alpha^a + m \Phi_{\mu,\nu}^a \Omega_\alpha^a - 2\tilde{m} \Omega_{\mu,\nu} \pi_\alpha + \tilde{m} f_{\mu,\nu} B_\alpha] + \\
&+ \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} \left[-\frac{M^2}{36} \Phi_\mu^{ac} \Phi_\nu^{bc} + \frac{M^2}{8} f_\mu^a f_\nu^b \right] + \frac{M^2}{2} \pi^a \pi_a
\end{aligned} \tag{22}$$

The gauge transformations take the form

$$\begin{aligned}
\delta_0 \Omega_\mu^{ab} &= D_\mu \eta^{ab} + \frac{m}{2} (e_\mu^{(a} \eta^{b)}) - \frac{2}{3} g^{ab} \eta_\mu - \frac{M^2}{36} \varepsilon_\mu^{c(a} \xi^{b)c} \\
\delta_0 \Phi_\mu^{ab} &= D_\mu \xi^{ab} - \varepsilon_\mu^{c(a} \eta^{b)c} + \frac{3m}{2} (e_\mu^{(a} \xi^{b)}) - \frac{2}{3} g^{ab} \xi_\mu \\
\delta_0 \Omega_\mu^a &= D_\mu \eta^a + 3m \eta_\mu^a + \frac{M^2}{4} \varepsilon_\mu^{ab} \xi^b \\
\delta_0 f_\mu^a &= D_\mu \xi^a + \varepsilon_\mu^{ab} \eta^b + m \xi_\mu^a \\
\delta_0 B^a &= -2\tilde{m} \eta^a, \quad \delta_0 \pi^a = \tilde{m} \xi^a
\end{aligned} \tag{23}$$

Having at our disposal the explicit form of gauge transformations we can build six gauge invariant objects (which we will call curvatures though there are two-forms as well as one-forms

among them)

$$\begin{aligned}
\mathcal{G}_{\mu\nu}{}^{ab} &= D_{[\mu}\Omega_{\nu]}{}^{ab} + \frac{m}{2}(e_{[\mu}{}^{(a}\Omega_{\nu]}{}^{b)}) + \frac{2}{3}g^{ab}\Omega_{[\mu,\nu]} - \frac{M^2}{36}\varepsilon_{[\mu}{}^c{}^{(a}\Phi_{\nu]}{}^{b)c} \\
\mathcal{H}_{\mu\nu}{}^{ab} &= D_{[\mu}\Phi_{\nu]}{}^{ab} - \varepsilon_{[\mu}{}^c{}^{(a}\Omega_{\nu]}{}^{b)c} + \frac{3m}{2}(e_{[\mu}{}^{(a}f_{\nu]}{}^{b)}) + \frac{2}{3}g^{ab}f_{[\mu,\nu]} \\
\mathcal{F}_{\mu\nu}{}^a &= D_{[\mu}\Omega_{\nu]}{}^a - 3m\Omega_{[\mu,\nu]}{}^a - \tilde{m}e_{[\mu}{}^{(a}B_{\nu]} + \frac{M^2}{4}\varepsilon_{[\mu}{}^{ab}f_{\nu]}{}^b \\
\mathcal{T}_{\mu\nu}{}^a &= D_{[\mu}f_{\nu]}{}^a + \varepsilon_{[\mu}{}^{ab}\Omega_{\nu]}{}^b - m\Phi_{[\mu,\nu]}{}^a + 2\tilde{m}e_{[\mu}{}^{(a}\pi_{\nu]} \\
\mathcal{B}_{\mu}{}^a &= D_{\mu}B^a + 2\tilde{m}\Omega_{\mu}{}^a - \frac{M^2}{2}\varepsilon_{\mu}{}^{ab}\pi^b - W_{\mu}{}^a \\
\Pi_{\mu}{}^a &= D_{\mu}\pi^a - \frac{1}{2}\varepsilon_{\mu}{}^{ab}B^b - \tilde{m}f_{\mu}{}^a - V_{\mu}{}^a
\end{aligned} \tag{24}$$

where zero form auxiliary fields W^{ab}, V^{ab} (that do not enter the free Lagrangian) are symmetric and transformed as follows

$$\delta W^{ab} = 6m\tilde{m}\eta^{ab}, \quad \delta V^{ab} = -m\tilde{m}\xi^{ab}$$

Then it is possible to rewrite the Lagrangian (22) using these curvatures in the following form

$$\mathcal{L}_0 = \frac{1}{4}\varepsilon^{\mu\nu\alpha}[\Omega_{\mu}{}^{ab}\mathcal{H}_{\nu\alpha}{}^{ab} + \Phi_{\mu}{}^{ab}\mathcal{G}_{\nu\alpha}{}^{ab} - \Omega_{\mu}{}^a\mathcal{T}_{\nu\alpha}{}^a - f_{\mu}{}^a\mathcal{F}_{\nu\alpha}{}^a - 2B_{\mu}\Pi_{\nu,\alpha} - 2\pi_{\mu}\mathcal{B}_{\nu,\alpha}] \tag{25}$$

Such description contains three symmetric pairs $(\Omega_{\mu}{}^{ab}, \Phi_{\mu}{}^{ab})$, $(\Omega_{\mu}{}^a, f_{\mu}{}^a)$ and (B^a, π^a) . Thus we can introduce new variables

$$\begin{aligned}
\hat{\Omega}_{\pm\mu}{}^{ab} &= \Omega_{\mu}{}^{ab} \pm \frac{M}{6}\Phi_{\mu}{}^{ab} & \hat{B}_{\pm}^a &= B^a \mp M\pi^a \\
\hat{\Omega}_{\pm\mu}{}^a &= \Omega_{\mu}{}^a \pm \frac{M}{2}f_{\mu}{}^a & \hat{W}_{\pm\mu}{}^a &= W_{\mu}{}^a \mp MV_{\mu}{}^a
\end{aligned}$$

while the corresponding curvatures will have the form

$$\begin{aligned}
\hat{\mathcal{G}}_{\pm\mu\nu}{}^{ab} &= D_{[\mu}\hat{\Omega}_{\pm\nu]}{}^{ab} + \frac{m}{2}[e_{[\mu}{}^{(a}\hat{\Omega}_{\pm\nu]}{}^{b)}] + \frac{2}{3}g^{ab}\hat{\Omega}_{\pm[\mu,\nu]} \mp \frac{M}{6}\varepsilon_{[\mu}{}^c{}^{(a}\hat{\Omega}_{\pm\nu]}{}^{b)c} \\
\hat{\mathcal{F}}_{\pm\mu\nu}{}^{ab} &= D_{[\mu}\hat{\Omega}_{\pm\nu]}{}^a - 3m\hat{\Omega}_{\pm[\mu,\nu]}{}^a - \tilde{m}e_{[\mu}{}^a\hat{B}_{\pm\nu]} \pm \frac{M}{2}\varepsilon_{[\mu}{}^{ab}\hat{\Omega}_{\pm\nu]}{}^b \\
\hat{\mathcal{B}}_{\pm\mu}{}^a &= D_{\mu}\hat{B}_{\pm}^a + 2\tilde{m}\hat{\Omega}_{\pm\mu}{}^a \pm \frac{M}{2}\varepsilon_{\mu}{}^{ab}\hat{B}_{\pm}^b - \hat{W}_{\pm\mu}{}^a
\end{aligned} \tag{26}$$

In terms of the new fields and curvatures the free Lagrangian can be rewritten as

$$\begin{aligned}
\mathcal{L}_0 &= \frac{1}{4M}\varepsilon^{\mu\nu\alpha}[3\hat{\Omega}_{+\mu}{}^{ab}\hat{\mathcal{G}}_{+\nu\alpha}{}^{ab} - \hat{\Omega}_{+\mu}{}^a\hat{\mathcal{F}}_{+\nu\alpha}{}^a + \hat{B}_{+\mu}\hat{\mathcal{B}}_{+\nu\alpha}] - \\
&\quad - \frac{1}{4M}\varepsilon^{\mu\nu\alpha}[3\hat{\Omega}_{-\mu}{}^{ab}\hat{\mathcal{G}}_{-\nu\alpha}{}^{ab} - \hat{\Omega}_{-\mu}{}^a\hat{\mathcal{F}}_{-\nu\alpha}{}^a + \hat{B}_{-\mu}\hat{\mathcal{B}}_{-\nu\alpha}]
\end{aligned} \tag{27}$$

We see that the variables are separated, i.e. now there is no mixing between them. Each half is invariant with respect to one type gauge transformations and it looks like the Chern-Simons

action for the massless fields. Moreover, as in the massless case the gauge invariance of the Lagrangian can be most easily checked using corresponding differential identities:

$$\begin{aligned}
D_{[\mu}\hat{\mathcal{G}}_{\pm\nu\alpha]}^{ab} &= -\frac{m}{2}[e_{[\mu}{}^{(a}\hat{\mathcal{F}}_{\pm\nu\alpha]}^{b)} + \frac{2}{3}g^{ab}\hat{\mathcal{F}}_{\pm[\mu\nu,\alpha]}] \pm \frac{M}{6}\varepsilon_{[\mu}{}^{c(a}\hat{\mathcal{G}}_{\pm\nu\alpha]}^{b)c} \\
D_{[\mu}\hat{\mathcal{F}}_{\nu\alpha]}^a &= -3m\hat{\mathcal{G}}_{\pm[\mu\nu,\alpha]}^a + \tilde{m}e_{[\mu}{}^a\hat{\mathcal{B}}_{\pm\nu,\alpha]} \mp \frac{M}{2}\varepsilon_{[\mu}{}^{ab}\hat{\mathcal{F}}_{\pm\nu\alpha]}^b \\
D_{[\mu}\hat{\mathcal{B}}_{\pm\nu]}^a &= 2\tilde{m}\hat{\mathcal{F}}_{\pm\mu\nu}^a \mp \frac{M}{2}\varepsilon_{[\mu}{}^{ab}\hat{\mathcal{B}}_{\pm\nu]}^b - \hat{\mathcal{W}}_{\pm\mu\nu}^a
\end{aligned} \tag{28}$$

Thus we can work with one half of the fields only, in this component form for the corresponding Lagrangian (with a plus sign omitted) looks like:

$$\begin{aligned}
\mathcal{L}_0 &= \frac{1}{4M}\{\varepsilon^{\mu\nu\alpha}[6\hat{\Omega}_\mu{}^{ab}D_\nu\hat{\Omega}_\alpha{}^{ab} - 2\hat{\Omega}_\mu{}^aD_\nu\hat{\Omega}_\alpha{}^a + \hat{B}_\mu D_\nu\hat{B}_\alpha + \\
&\quad + 12m\hat{\Omega}_{\mu,\nu}{}^a\hat{\Omega}_\alpha{}^a + 4\tilde{m}\hat{\Omega}_{\mu,\nu}\hat{B}_\alpha] - \\
&\quad - 2M\{\frac{\mu\nu}{ab}\}\hat{\Omega}_\mu{}^{ac}\hat{\Omega}_\nu{}^{bc} + M\{\frac{\mu\nu}{ab}\}\hat{\Omega}_\mu{}^a\hat{\Omega}_\nu{}^b + M\hat{B}^2\}
\end{aligned} \tag{29}$$

This Lagrangian is invariant under the following gauge transformations

$$\begin{aligned}
\delta_0\hat{\Omega}_\mu{}^{ab} &= D_\mu\hat{\eta}^{ab} + \frac{m}{2}[e_\mu{}^{(a}\hat{\eta}^{b)} - \frac{2}{3}g^{ab}\hat{\eta}_\mu] - \frac{M}{6}\varepsilon_\mu{}^{c(a}\hat{\eta}^{b)c} \\
\delta_0\hat{\Omega}_\mu{}^a &= D_\mu\hat{\eta}^a + \frac{M}{2}\varepsilon_\mu{}^{ab}\hat{\eta}^b + 3m\hat{\eta}_\mu{}^a, \quad \delta_0\hat{B}^a = -2\tilde{m}\hat{\eta}^a
\end{aligned} \tag{30}$$

where the gauge parameters are defined as follows

$$\hat{\eta}^{ab} = \eta^{ab} + \frac{M}{6}\xi^{ab}, \quad \hat{\eta}^a = \eta^a + \frac{M}{2}\xi^a$$

Thus we have obtained a frame-like gauge invariant description of massive spin 3 fields in terms of separated variables. Actually we have two identical copies of the same Lagrangian. Therefore it is sufficient to study a structure only one of them and restore a total Lagrangian at the very end. Now we are ready for generalization to the arbitrary spin field case. We will see that the similar situation is also valid in general case.

4 Massive field with arbitrary integer spin

In this section we construct frame-like gauge invariant formulation for massive fields with the arbitrary integer spin. Having gained some experience from the spin 3 case, from the very beginning we will work in terms of separated variables. Thus we will look for the Lagrangian in the form

$$\mathcal{L}_0 = \sum_{k=1}^{s-1} (-1)^k \frac{a_k}{2} \varepsilon^{\mu\nu\alpha} \hat{\Omega}_\mu{}^{(k)} \hat{\mathcal{G}}_{\nu\alpha}{}^{(k)} + \frac{a_0}{2} \varepsilon^{\mu\nu\alpha} \hat{B}_\mu \hat{\mathcal{B}}_{\nu,\alpha} \tag{31}$$

while for the curvatures we will use the following expressions generalizing that of (26):

$$\begin{aligned}
\hat{\mathcal{G}}_{\mu\nu}{}^{(k)} &= D_{[\mu}\hat{\Omega}_{\nu]}^{(k)} - \gamma_k \hat{\Omega}_{[\mu,\nu]}^{(k)} + \beta_k \varepsilon_{[\mu}{}^{b(1}\hat{\Omega}_{\nu]}^{k-1)b} + \\
&\quad + \alpha_k [e_{[\mu}{}^{(1}\hat{\Omega}_{\nu]}^{k-1)} + \frac{2}{2k-1}g^{(2}\hat{\Omega}_{[\mu,\nu]}^{k-2)}] \\
\hat{\mathcal{B}}_\mu{}^a &= D_\mu\hat{B}^a - \gamma_0\hat{\Omega}_\mu{}^a + \beta_1\varepsilon_\mu{}^{ba}\hat{B}^b - \hat{W}_\mu{}^a
\end{aligned} \tag{32}$$

In this, corresponding gauge transformations will look like (compare (30)):

$$\begin{aligned}\delta_0 \hat{\Omega}_\mu^{(k)} &= D_\mu \hat{\eta}^{(k)} + \alpha_k [e_\mu^{(1)} \hat{\eta}^{k-1} - \frac{2}{2k-1} g^{(2)} \hat{\eta}_\mu^{k-2}] + \\ &\quad + \beta_k \varepsilon_\mu^{b(1)} \hat{\eta}^{k-1b} + \gamma_k \hat{\eta}_\mu^{(k)} \\ \delta \hat{B}^a &= \gamma_0 \hat{\eta}^a\end{aligned}\tag{33}$$

First of all, note that in order to the Lagrangian rewritten in terms of initial fields $\hat{\Omega}_\mu^{(k)} = \Omega_\mu^{(k)} - \beta_k \Phi_\mu^{(k)}$ and $\hat{B}^a = B^a + \kappa_0 \pi^a$ had the canonically normalized kinetic terms, we have to put

$$a_k = -\frac{1}{4\beta_k}, \quad k \geq 1, \quad a_0 = -\frac{1}{4\kappa_0}, \quad \kappa_0 = 2\beta_1\tag{34}$$

Then one can show by direct calculations that for curvatures (32) be invariant under gauge transformations (33) the following two relations must hold

$$(k+2)\beta_{k+1} = k\beta_k\tag{35}$$

$$\frac{2k+3}{2k+1} \gamma_k \alpha_{k+1} - \gamma_{k-1} \alpha_k + \beta_k^2 - \lambda^2 = 0\tag{36}$$

From the explicit expressions for curvatures it is not hard to obtain the differential identities

$$\begin{aligned}D_{[\mu} \hat{\mathcal{G}}_{\nu\alpha]}^{(k)} &= -\beta_k \varepsilon_{[\mu}^{a(1)} \hat{\mathcal{G}}_{\nu\alpha]}^{k-1a} - \gamma_k \hat{\mathcal{G}}_{[\mu\nu,\alpha]}^{(k)} - \alpha_k [e_{[\mu}^{(1)} \hat{\mathcal{G}}_{\nu\alpha]}^{k-1} + \frac{2}{2k-1} g^{(2)} \hat{\mathcal{G}}_{[\mu\nu,\alpha]}^{k-2}] \\ D_{[\mu} \hat{\mathcal{B}}_{\nu]}^a &= -\beta_1 \varepsilon_{[\mu}^{ba} \hat{\mathcal{B}}_{\nu]}^b - \gamma_0 \hat{\mathcal{G}}_{\mu\nu}^a\end{aligned}\tag{37}$$

Using them one can easily check that the Lagrangian will be gauge invariant provided one more relation holds

$$ka_k \alpha_k - a_{k-1} \gamma_{k-1} = 0\tag{38}$$

Now let us solve these relations to obtain expressions for all unknown coefficients. Equation (35) provides recurrent relation on β_k that allows expressing all of them in terms of β_{s-1}

$$\beta_k = \frac{s(s-1)}{k(k+1)} \beta_{s-1}\tag{39}$$

Then from (38) we obtain

$$\gamma_{k-1} = \frac{k(k+1)}{(k-1)} \alpha_k, \quad k \geq 2, \quad \gamma_0 = 2\alpha_1\tag{40}$$

Substituting all these in (35), (36) we get:

$$\frac{(2k+3)(k+1)(k+2)}{k(2k+1)} \alpha_{k+1}^2 - \frac{k(k+1)}{(k-1)} \alpha_k^2 + \beta_k^2 - \lambda^2 = 0, \quad k \geq 2\tag{41}$$

$$10\alpha_2^2 - 2\alpha_1^2 + \beta_1^2 - \lambda^2 = 0\tag{42}$$

Taking into account that $\alpha_s = 0$ equation (41) for $k = s-1$ gives us

$$\beta_{s-1}^2 = \frac{s(s-1)}{(s-2)} \alpha_{s-1}^2 + \lambda^2\tag{43}$$

while for other values of k it can be considered as recurrent relation on α_k . This relation can be directly solved and we obtain

$$\alpha_k = \frac{(k-1)(s-k)(s+k)}{k^3(k+1)(2k+1)} [\hat{\alpha}_{s-1}^2 + (s-k-1)(s+k-1)\lambda^2], \quad k > 1 \quad (44)$$

where we have introduced

$$\hat{\alpha}_{s-1}^2 = \frac{s(s-1)^3}{s-2} \alpha_{s-1}^2$$

while the remaining unknown coefficient α_1 is determined by (42).

The resulting Lagrangian can be written in component terms as follows (compare (29)):

$$\begin{aligned} \mathcal{L} = & \sum_{k=1}^{s-1} \frac{(-1)^{k+1}}{4\beta_k} [\varepsilon^{\mu\nu\alpha} (\hat{\Omega}_\mu^{(k)} D_\nu \hat{\Omega}_\alpha^{(k)} + 2k\alpha_k \hat{\Omega}_{\mu,\nu}^{(k-1)} \hat{\Omega}_\alpha^{(k-1)}) + k\beta_k \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} \hat{\Omega}_\mu^{a(k-1)} \hat{\Omega}_\nu^{b(k-1)}] - \\ & - \frac{1}{8\beta_1} [\varepsilon^{\mu\nu\alpha} \hat{B}_\mu D_\nu \hat{B}_\alpha - 2\beta_1 \hat{B}^a \hat{B}^a] \end{aligned}$$

Now we can easily obtain the formulation in terms of initial variables $\hat{\Omega}_\mu^{(k)} = \Omega_\mu^{(k)} - \beta_k \Phi_\mu^{(k)}$ and $\hat{B}^a = B^a + 2\beta_1 \pi^a$. As we have already noted, the Lagrangian will have a canonical form:

$$\mathcal{L} = \sum_{k=0}^{s-1} (-1)^k \frac{1}{4} \varepsilon^{\mu\nu\alpha} [\Omega_\mu^{(k)} \mathcal{H}_{\nu\alpha}^{(k)} + \Phi_\mu^{(k)} \mathcal{G}_{\nu\alpha}^{(k)}] - \frac{1}{2} \varepsilon^{\mu\nu\alpha} [B_\mu \Pi_{\nu\alpha} + \pi_\mu \mathcal{B}_{\nu\alpha}] \quad (45)$$

where gauge invariant curvatures are defined as follows

$$\begin{aligned} \mathcal{G}_{\mu\nu}^{(k)} &= D_{[\mu} \Omega_{\nu]}^{(k)} - \frac{(k+1)(k+2)\alpha_k}{k} \Omega_{[\mu,\nu]}^{(k)} - \beta_k^2 \varepsilon_{[\mu}^{a(1)} \Phi_{\nu]}^{k-1)a} + \\ &+ \alpha_k [e_{[\mu}^{(1)} \Omega_{\nu]}^{k-1)} + \frac{2}{2k-1} g^{(2)} \Omega_{[\mu,\nu]}^{k-2)}] \\ \mathcal{H}_{\mu\nu}^{(k)} &= D_{[\mu} \Phi_{\nu]}^{(k)} - (k+1)\alpha_{k+1} \Phi_{[\mu,\nu]}^{(k)} - \varepsilon_{[\mu}^{a(1)} \Omega_{\nu]}^{k-1)a} + \\ &+ \frac{\gamma_{k-1}}{k} [e_{[\mu}^{(1)} \Phi_{\nu]}^{k-1)} + \frac{2}{2k-1} g^{(2)} \Phi_{[\mu,\nu]}^{k-2)}] \\ \mathcal{B}_\mu^a &= D_\mu B^a - 2\alpha_1 \Omega_\mu^a + 2\beta_1^2 \varepsilon_\mu^{ba} \pi^b - W_\mu^a \\ \Pi_\mu^a &= D_\mu \pi^a + \alpha_1 f_\mu^a + \frac{1}{2} \varepsilon_\mu^{ba} B^b - V_\mu^a \end{aligned} \quad (46)$$

while gauge transformations leaving the Lagrangian and curvatures invariant look like

$$\begin{aligned} \delta \Omega_\mu^{(k)} &= D_\mu \eta^{(k)} + \alpha_k [e_\mu^{(1)} \eta^{k-1)} - \frac{2}{3} g^{(2)} \eta_\mu^{k-2)}] + \\ &- \beta_k^2 \varepsilon_\mu^{b(1)} \xi^{k-1)b} + \frac{(k+1)(k+2)}{k} \alpha_k \eta_\mu^{(k)} \\ \delta \Phi_\mu^{(k)} &= D_\mu \xi^{(k)} + \frac{\gamma_{k-1}}{k} [e_\mu^{(1)} \xi^{k-1)} - \frac{2}{3} g^{(2)} \xi_\mu^{k-2)}] - \\ &- \varepsilon_\mu^{a(1)} \eta^{k-1)a} + (k+1)\alpha_k \xi_\mu^{(k)} \\ \delta B^a &= 2\alpha_1 \eta^a, \quad \delta \pi^a = -\alpha_1 \xi^a \end{aligned} \quad (47)$$

Let us also give here a component form for the Lagrangian to be compared with (29)

$$\begin{aligned}
\mathcal{L} = & \sum_{k=1}^{s-1} (-1)^{k+1} \left[\frac{k}{2} \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} \Omega_\mu^{a(k-1)} \Omega_\alpha^{b(k-1)} - \varepsilon^{\mu\nu\alpha} \Omega_\mu^{(k)} D_\nu \Phi_\alpha^{(k)} + \right. \\
& \left. + \frac{k}{2} \beta_k^2 \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} \Phi_\mu^{a(k-1)} \Phi_\alpha^{b(k-1)} \right] - \\
& - \sum_{k=2}^{s-1} (-1)^{k+1} \varepsilon^{\mu\nu\alpha} [\gamma_{k-1} \Omega_{\mu,\nu}^{(k-1)} \Phi_\alpha^{(k-1)} + \alpha_k k \Omega_\mu^{(k-1)} \Phi_{\nu,\alpha}^{(k-1)}] + \\
& + \frac{1}{2} B^a B^a - \varepsilon^{\mu\nu\alpha} B_\mu D_\nu \pi_\alpha - \varepsilon^{\mu\nu\alpha} [\alpha_1 B_\mu f_{\nu,\alpha} - 2\alpha_1 \Omega_{\mu,\nu} \pi_\alpha] + 2\beta_1^2 \pi^a \pi^a \quad (48)
\end{aligned}$$

As in terms of separated variables, there is one arbitrary parameter α_{s-1} which is fixed by the normalization of the mass. For example, the canonical normalization corresponds to

$$\alpha_{s-1}^2 = \frac{s-2}{s(s-1)^2} m^2$$

Summary

In this paper we have given the gauge invariant Lagrangian formulation of $d = 3$ massive bosonic higher spin fields in $(A)dS_3$ space, working within the frame-like approach. The gauge invariance is ensured by the use of the Stueckelberg auxiliary fields. Similarly to the massless case we have rewritten the Lagrangians for massive fields in terms of gauge invariant objects similarly to Chern-Simons actions. This is achieved by a partial gauge fixing, so that the scalar field is excluded. This was clearly demonstrated for massive spin 3 field example with a further reformulation of the theory in terms of separated field variables. The separated variables allow us working with one half of fields that greatly simplifies all calculations. Finally, using the separated variables, we have constructed the Lagrangian for the massive field of arbitrary integer spin and have shown how to restore the theory in terms of original fields. Due to the partial gauge fixing, the resulting theory in original fields does not contain the auxiliary scalar field.

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